Practice 2

Carlo Cavicchia

ccavicchia@luiss.it

Probability: events

- Combinations and permutations
- Mutually exclusive events
- Conditional probability law
- Independent events
- Bayes Theorem

Exercise 1

Let us consider two coins: M_1 that gives "head" with probability equal to 0.8 and M_2 that gives "head" with probability equal to 0.2. Chosen one coin, flip it three times.

Let us consider X the random variable counting the number of "head".

Exercise 1

Let us consider two coins: M_1 that gives “head” with probability equal to 0.8 and M_2 that gives “head” with probability equal to 0.2. Chosen one coin, flip it three times.

Let us consider X the random variable counting the number of “head”.

- What is the probability of having “head” at the first flip?

Exercise 1

Let us consider two coins: M_1 that gives “head” with probability equal to 0.8 and M_2 that gives “head” with probability equal to 0.2. Chosen one coin, flip it three times.

Let us consider X the random variable counting the number of “head”.

- What is the probability of having “head” at the first flip?
- What is the probability of having “head” at the second flip?

Exercise 1

Let us consider two coins: M_1 that gives "head" with probability equal to 0.8 and M_2 that gives "head" with probability equal to 0.2. Chosen one coin, flip it three times.

Let us consider X the random variable counting the number of "head".

› What is the probability of having "head" at the first flip?

› What is the probability of having "head" at the second flip?

› Are they indipendent?

Exercise 1

Let us consider two coins: M_1 that gives "head" with probability equal to 0.8 and M_2 that gives "head" with probability equal to 0.2. Chosen one coin, flip it three times.

Let us consider X the random variable counting the number of "head".

- What is the probability of having "head" at the first flip?
- What is the probability of having "head" at the second flip?
- Are they indipendent?
- Find the distribution of X.

Exercise 1

Let us consider two coins: M_1 that gives "head" with probability equal to 0.8 and M_2 that gives "head" with probability equal to 0.2. Chosen one coin, flip it three times.

Let us consider X the random variable counting the number of "head".

- What is the probability of having "head" at the first flip?
- What is the probability of having "head" at the second flip?
- Are they indipendent?
- Find the distribution of X.
- Given that the number of "head" is equal to one, which coin is most likely to have been flipped?

Exercise 2

Let us consider a box containing three different coins. The first coin has a “head” on both the faces, the second coin has a “tail” on both the faces, the third coin is a standard one: “head” on one face and “tail” on the other.

We pick one coin at random from the box.

Exercise 2

Let us consider a box containing three different coins. The first coin has a "head" on both the faces, the second coin has a "tail" on both the faces, the third coin is a standard one: "head" on one face and "tail" on the other.

We pick one coin at random from the box.

- What is the probability of the front face to be a "head"?

Exercise 2

Let us consider a box containing three different coins. The first coin has a "head" on both the faces, the second coin has a "tail" on both the faces, the third coin is a standard one: "head" on one face and "tail" on the other.

We pick one coin at random from the box.

- What is the probability of the front face to be a "head"?
- Given that the front face shows a "head", what is the probability of the back face to be a "head" as well??

Probability: random variables

- Discrete random variables: pmf and cdf
- Famous discrete: Bernoulli, Binomial, Poisson ...
- Continuous random variables: pdf and cdf
- Famous continuous: Uniform, Exponential, Normal ...

Exercise 3

Let us consider the following function:

$$p(x) = \begin{cases} 0.0 & X < 1 \\ 0.1 & 1 \leq X < 2 \\ 0.8 & 2 \leq X < 3 \\ 1.0 & X \geq 3 \end{cases}$$

- Is it a valid pmf?

Exercise 3

Let us consider the following function:

$$p(x) = \begin{cases} 0.0 & X < 1 \\ 0.1 & 1 \leq X < 2 \\ 0.8 & 2 \leq X < 3 \\ 1.0 & X \geq 3 \end{cases}$$

- Is it a valid pmf?
- Is it a valid cdf?

Exercise 3

Let us consider the following function:

$$p(x) = \begin{cases} 0.0 & X < 1 \\ 0.1 & 1 \leq X < 2 \\ 0.8 & 2 \leq X < 3 \\ 1.0 & X \geq 3 \end{cases}$$

- Is it a valid pmf?
- Is it a valid cdf?
- Can you derive the pmf from the cdf?

Exercise 3

Let us consider the following function:

$$p(x) = \begin{cases} 0.0 & X < 1 \\ 0.1 & 1 \leq X < 2 \\ 0.8 & 2 \leq X < 3 \\ 1.0 & X \geq 3 \end{cases}$$

- Is it a valid pmf?
- Is it a valid cdf?
- Can you derive the pmf from the cdf?
- Plot them

Exercise 3

Let us consider the following function:

$$p(x) = \begin{cases} 0.0 & X < 1 \\ 0.1 & 1 \leq X < 2 \\ 0.8 & 2 \leq X < 3 \\ 1.0 & X \geq 3 \end{cases}$$

- Is it a valid pmf?
- Is it a valid cdf?
- Can you derive the pmf from the cdf?
- Plot them
- What is the probability that $0.5 < X < 3$? And $2 \leq X \leq 4$?

Exercise 4

T describes the (continuous) time interval (in years) intercurring between two lightnings hitting the ground in the same area. Waiting times are usually assumed to be distributed according to a distribution with density:

$$f_T(t;\lambda) = \begin{cases} \lambda e^{-\lambda t} & t \geq 0 \\ 0 & t < 0 \end{cases}; \quad \lambda > 0$$

A scientist keeps his camera pointed toward an area with the aim to record one lightning.

Exercise 4

T describes the (continuous) time interval (in years) intercurring between two lightnings hitting the ground in the same area. Waiting times are usually assumed to be distributed according to a distribution with density:

$$f_T(t;\lambda) = \begin{cases} \lambda e^{-\lambda t} & t \geq 0 \\ 0 & t < 0 \end{cases}; \quad \lambda > 0$$

A scientist keeps his camera pointed toward an area with the aim to record one lightning.

- Prove that $f_T(t;\lambda)$ is a valid pdf.

Exercise 4

T describes the (continuous) time interval (in years) intercurring between two lightnings hitting the ground in the same area. Waiting times are usually assumed to be distributed according to a distribution with density:

$$f_T(t;\lambda) = \begin{cases} \lambda e^{-\lambda t} & t \geq 0 \\ 0 & t < 0 \end{cases}; \quad \lambda > 0$$

A scientist keeps his camera pointed toward an area with the aim to record one lightning.

- Prove that $f_T(t;\lambda)$ is a valid pdf.
- What is the probability that he will record the first lightning within t^* years?

Exercise 4

T describes the (continuous) time interval (in years) intercurring between two lightnings hitting the ground in the same area. Waiting times are usually assumed to be distributed according to a distribution with density:

$$f_T(t;\lambda) = \begin{cases} \lambda e^{-\lambda t} & t \geq 0 \\ 0 & t < 0 \end{cases}; \quad \lambda > 0$$

A scientist keeps his camera pointed toward an area with the aim to record one lightning.

- Prove that $f_T(t;\lambda)$ is a valid pdf.
- What is the probability that he will record the first lightning within t^* years?
- Assuming that $\lambda = 0.1$ and $t^* = 100$, compute this probability.

Exercise 4

T describes the (continuous) time interval (in years) intercurring between two lightnings hitting the ground in the same area. Waiting times are usually assumed to be distributed according to a distribution with density:

$$f_T(t;\lambda) = \begin{cases} \lambda e^{-\lambda t} & t \geq 0 \\ 0 & t < 0 \end{cases}; \quad \lambda > 0$$

A scientist keeps his camera pointed toward an area with the aim to record one lightning.

- Prove that $f_T(t;\lambda)$ is a valid pdf.
- What is the probability that he will record the first lightning within t^* years?
- Assuming that $\lambda = 0.1$ and $t^* = 100$, compute this probability.
- Given that he has already waited t_0 years, what is the probability that he will record the first lightning within additional t^* years?